

EE 330 HW 8 Solutions
Spring 2024

Problem 1

Both M_1 and M_2 are in saturation as $V_{GS} = V_{DS}$

$$\Rightarrow I_{D1} = I_{D2}$$

$$\Rightarrow I_{D2} = \frac{100 \times 10^{-6}}{2} \times \frac{4}{2} \times (5 - V_{out} - 0.75)^2$$

$$\Rightarrow I_{D1} = \frac{100 \times 10^{-6}}{2} \times \frac{5}{2} \times (V_{out} - 0 - 0.75)^2$$

$$\Rightarrow 100 \times 10^{-6} (4.25 - V_{out})^2 = 1.25 \times 10^{-4} (V_{out} - 0.75)^2$$

$$\Rightarrow (4.25 - V_{out})^2 = 1.25 (V_{out} - 0.75)^2$$

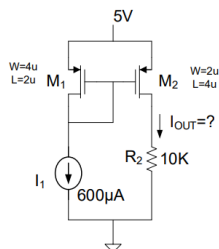
$$0.25 V_{out}^2 + 6.625 V_{out} - 17.36 = 0$$

$$V_{out} = 2.4 \text{ V}, \quad -28.9 \text{ V}$$

$\Rightarrow V_{out} = -28.9 \text{ V}$ puts M_1 in cut off

Thus, $V_{out} = 2.4 \text{ V}$

Problem 2

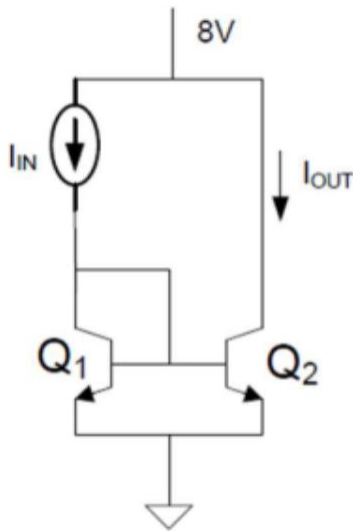


$$I_{D1} = \frac{\mu_p C_{OX} W_1}{2L_1} (V_{GS1} - V_{THp})^2$$

$$I_{D2} = \frac{\mu_p C_{OX} W_2}{2L_2} (V_{GS2} - V_{THp})^2$$

Since $V_{GS1} = V_{GS2}$, taking the ratio I_{D2}/I_{D1} , we obtain $\frac{I_{D2}}{I_{D1}} = \frac{W_2}{L_2} \frac{L_1}{W_1} = \frac{1}{4}$ so $I_{D2} = 150 \mu A$

Problem 3



Since $V_{BE1} = V_{BE2}$, can write the two equations

$$I_{IN} = I_{C1} + I_{B1} + I_{B2} = J_S A_{E1} e^{\frac{nV_{BE1}}{V_t}} + J_S \frac{A_{E1}}{\beta_n} e^{\frac{nV_{BE1}}{V_t}} + J_S \frac{A_{E2}}{\beta_n} e^{\frac{nV_{BE1}}{V_t}}$$

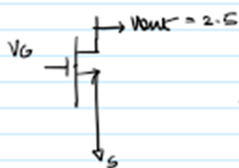
$$I_{OUT} = I_{C2} = J_S A_{E2} e^{\frac{nV_{BE1}}{V_t}}$$

Taking the ratio, the J_S and exponential terms cancel and we obtain

$$\frac{I_{OUT}}{I_{IN}} = \frac{A_{E2}}{A_{E1} + \frac{A_{E1}}{\beta_n} + \frac{A_{E2}}{\beta_n}} = \frac{600}{100 + \frac{100 + 600}{100}} = 5.61$$

Since $I_{IN} = 1 \text{ mA}$, it follows that $I_{OUT} = 5.61 \text{ mA}$

Problem 4.



Assume NMOS in saturation

$$\therefore V_{ds} = V_{gs} - V_{th} \quad \{ \text{let } V_{th} = 0.75 \}$$

$$V_{out} = V_g - 0.75$$

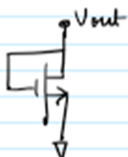
$$V_g = 2.5 + 0.75 = 3.25 > V_T$$

But we only have 5V supply

Let's make $V_g = V_{out}$

For saturation $V_{ds} \geq V_{gs} - V_{th}$

$$V_{out} \geq V_{out} - V_{th} \quad (\text{which is valid})$$



Let's provide another NMOS



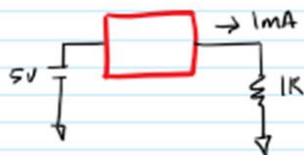
current through M2

$$I_D = \frac{\mu_n C_{ox}}{2} \frac{W_2}{L_2} (5 - 2.5 - 0.75)^2 = \frac{\mu_n C_{ox}}{2} \times \frac{W_1}{L_1} (2.5 - 0.75)^2$$

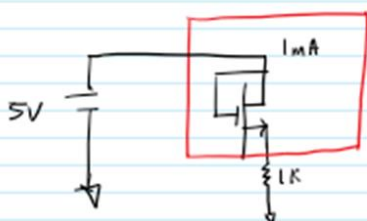
$$\frac{W_2}{L_2} = \frac{W_1}{L_1}$$

Let $L_1 = L_2 = 2\mu$ and $W_1 = W_2 = 4\mu$

Problem 5.



$$V_{out} = 1\text{mA} \times 1\text{k} = 1\text{V}$$

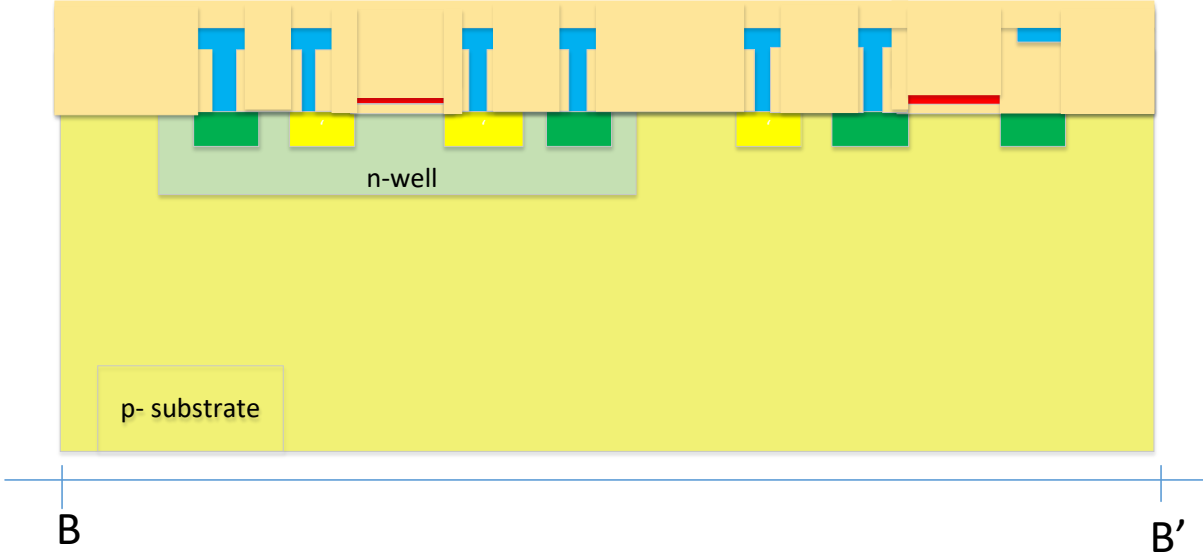


$$I_{Df} = \frac{\mu_n C_{ox}}{2} \frac{W_1}{L_1} (5 - 1 - 0.75)^2 = 1 \times 10^{-3}$$

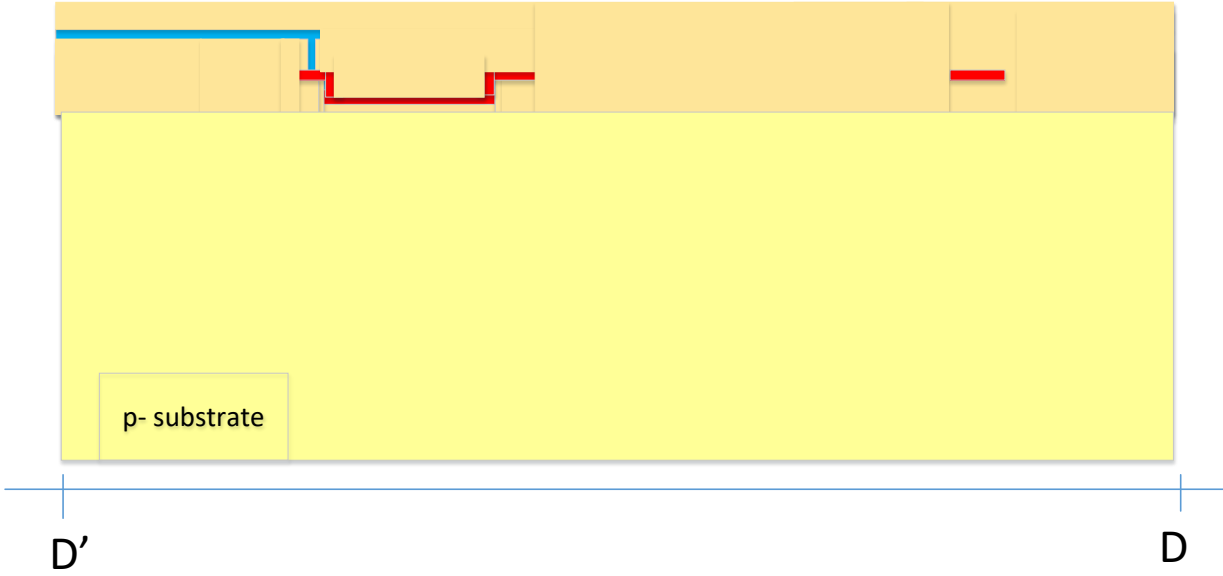
$$\frac{W_1}{L_1} = \frac{1 \times 10^{-3}}{50 \times 10^{-6} \times 10.6} = 1.9$$

$$W = 3.8 \mu\text{m} \quad L = 2 \mu\text{m}$$

Problem 6



Problem 7

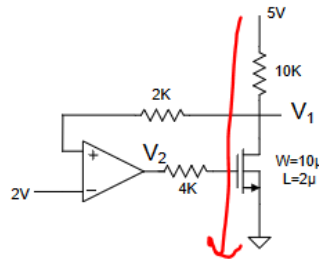


Problem 8

- a) Assuming $V_{BE}=0.6V$, we obtain $I_B = \frac{9.4V}{600K}$. Thus $I_C = \beta I_B = 100 \frac{9.4}{600K} = 1.57mA$ and thus
- $$V_{OUT} = 10 - I_C \cdot 2.5K = 6.1V$$
- b) Replacing the $\beta=100$ with $\beta=50$ in part a) we obtain $I_C=0.78mA$ and $V_{OUT}=8.04V$
- c) The solution is independent of β so the outputs will not change significantly
- d) The solution is also independent of β so the outputs will not change significantly

Problem 9

With the op amp being ideal, we can assume that there is no current flowing into either device input. This means that there is no current flowing through the $2k\Omega$ or the $4k\Omega$ resistor and thus $V_1=2V$.



Since $V_1=2V$, it follows that $I_D = \frac{5V-2V}{10000\Omega} = 0.3mA$. Since the MOSFET is operating in saturation, it thus follows that

$$I_{DS} = \frac{\mu C_{ox} W}{2L} (V_{GS} - V_T)^2$$

$$0.3mA = \frac{\left(100 \frac{\mu A}{V^2}\right) (10\mu m)}{2(2\mu m)} (V_{GS} - 0.75V)^2$$

$$1.09545V = V_{GS} - 0.75V$$

This gives us a $V_{GS} = V_2 = 1.84545V$. Note this solution satisfies both requirements for saturation ($V_{GS} \geq V_T$, $V_{DS} \geq V_{GS} - V_T$).

Problem 10.

To keep M1 in saturation (since $V_{GS} > V_{TH}$), must have $V_{DS} > V_{GS} - V_{TH}$. Substituting values into this equation we obtain $4V - I_D R - -2V \geq 2V - 0.5V$. This can be expressed as $R \leq \frac{4.5V}{I_D}$. It remains

to find I_D . But $I_D = \frac{\mu_n C_{ox} W}{2L} (V_{GS} - V_{TH})^2$. Substituting in the model parameters given with $V_{GS}=2V$, obtain $I_D=0.1875mA$. Thus $R \leq \frac{4.5V}{0.1875mA} = 24K\Omega$